CLAIMS

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1. A method of calculating the fast Fourier transform or the inverse fast Fourier transform of a digital signal defined by a series of N real starting samples x(n), with N a power of two and $n \in [0..N-1]$, comprising successive transformation steps (2) for transforming input samples into output samples, all the transformation steps being performed by means of a single set of butterflies with several inputs and several outputs, the operating mode of which is modified selectively in each transformation step, the input and output samples of each transformation step being stored in a storage memory, a series of N output samples y(n) representative of the fast Fourier transform or the inverse fast Fourier transform of the output samples x(n) being provided in the last transformation step,

characterized in that output samples y(n) are real,

and in that the output samples of a butterfly replace the corresponding input samples of the same rank in the storage memory, so that, if the starting samples x(n) processed in the first transformation step are classified in bit-reversed order of their index n, output samples y(n) are provided in the last transformation step in ascending order of index n, these output samples being defined by the following relations:

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$y(0) = Re[X(0)]$

$y(n) = Re[X((n+1)/2)]$ for n being odd and different from N-1

$y(n) = Im[X(n/2)]$ for n being even and different from 0

$y(N-1) = Re[X(N/2)]$

where samples $X(n)$, with $n \in [0..N-1]$, designate the complex samples of the series corresponding to the fast or inverse fast Fourier transform of the starting sample series $x(n)$.

2. A method of calculating the fast Fourier transform or the inverse fast Fourier transform of a digital signal defined by a series of N complex samples $X(n)$ conjugated by pairs represented by a series of N real starting samples $y(n)$, with N power of two and $n \in [0..N-1]$, the starting samples $y(n)$ being defined as follows:

$y(0) = Re[X(0)]$

$y(n) = Re[X((n+1)/2)]$ for n being odd and different from N-1

$y(n) = Im[X(n/2)]$ for n being even and different from 0

$y(N-1) = Re[X(N/2)]$

this calculation method comprising successive transformation steps for transforming input samples into output samples, a series of N output samples $x(n)$ representative of this fast or inverse fast Fourier transform being provided in the last transformation step, all the transformation steps being performed by means of a single set of butterflies with several inputs and several outputs, the operating mode of which

is modified selectively in each transformation step, the input and output samples of each transformation step being stored in a storage memory,

characterized in that output samples x(n) are real,

and in that the output samples of a butterfly replace the corresponding input samples of the same rank in the storage memory, so that, if the starting samples y(n) processed in the first transformation step are classified in ascending order of index n, the output samples x(n) are provided in the last transformation step in bit-reversed order of index n.

3. The calculation method according to claim 1 or 2, characterized in that, in each transformation step, each butterfly transforms input sample pairs, the ranks of the input samples of the same pair within the series of input samples of said transformation step being symmetrical with respect to a center between the end rank values of the input samples transformed by said butterfly.

4. The calculation method according to claim 3, characterized in that it comprises μ-1 transformation steps E_p with $\mu=\log_2(N)$ and $p \in [0..\mu-2]$.

5. The calculation method according to claim 4, in turn dependent on claim 3, in turn dependent on claim 1, characterized in further comprising:

- a preliminary step of modifying the sequence of the starting samples x(n) ranked in ascending order of index n and showing them in bit-reversed order of index n in the first transformation step, and

– a final step of processing the series of output samples y(n) and providing a series of N complex conjugated samples X(n) corresponding to the fast or the inverse fast Fourier transform of the series of starting samples x(n).

6. The calculation method of claim 4, in turn dependent on claim 3, in turn dependent on claim 1, or according to claim 5, characterized in that, in each transformation step E_p, butterflies are distributed among $N/2^{p+2}$ calculation blocks,

in that each calculation block has a peripheral butterfly and/or 2^p-1 internal butterflies,

in that the peripheral butterfly of the rank α calculation block in transformation step E_β transforms the input samples of rank $2^{\beta+2}\alpha$, $2^{\beta+2}\alpha+2^{\beta+1}-1$, $2^{\beta+2}\alpha+2^{\beta+1}$, $2^{\beta+2}\alpha+2^{\beta+2}-1$ into output samples of the same rank,

and in that the internal rank τ butterfly of the rank α calculation block in transformation step E_β transforms the input samples of rank $2^{\beta+2}\alpha+2\tau+1$, $2^{\beta+2}\alpha+2\tau+2$, $2^{\beta+2}\alpha+2^{\beta+2}-2\tau-3$, $2^{\beta+2}\alpha+2^{\beta+2}-2\tau-2$ into output samples of the same rank, with $\beta\geq1$.

7. The calculation method according to claim 6, characterized in that each butterfly is assigned a coefficient W^s, whereon the calculation inside the butterfly is based, said coefficient being equal to $e^{-j(2\pi s/N)}$ with $s \in [0..N/4-1]$ for a fast Fourier transform and is equal to $e^{j(2\pi s/N)}$ with $s \in [0..N/4-1]$ for an inverse fast Fourier transform.

8. Calculation method according to claim 7, characterized in that the internal rank τ butterfly of

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the rank α calculation block in transformation step E_β is assigned coefficient W^δ with $\delta = (\tau+1).(N/2^{\beta+2})$.

9. The calculation method according to claim 8, characterized in that the butterflies for implementing the transformation steps are all of the same type and have

- four inputs for receiving input samples and four outputs for providing output samples,
- four additional inputs, respectively primary mode, secondary mode, permutation, and coefficient inputs,

in order to selectively apply different transformation operations to the input samples, each operation being determined by the values assigned to the primary mode, secondary mode, permutation signals, and a coefficient admitted on said corresponding additional inputs.

10. The calculation method according to claim 9, characterized in that, for each butterfly, the primary mode signal is 0 for a peripheral butterfly and 1 for an internal butterfly,

in that the permutation signal is 0 for the even rank calculation blocks, including rank 0, and 1 for the other ones.

11. The calculation method according to claim 10, characterized in that, in transformation step Ep, each calculation block comprises one peripheral butterfly and 2^p-1 internal butterflies.

12. The calculation method according to claim 11, characterized in that the secondary mode signal is 1 if

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the peripheral butterfly is used for the last transformation step, and otherwise 0.

13. The calculation method according to claim 12, characterized in that, for four input samples e1, e2, e3, and e4, and for a complex coefficient $W^s=A+j.B$, the butterfly delivers the following output samples s1, s2, s3, and s4:

1) if the primary mode and secondary mode signals are 0:

s1 = e1 + e2
s2 = e1 - e2
s3 = e4 - e3
s4 = e3 + e4

2) if the primary mode signal is 0 and the secondary mode signal is 1:

s1 = e1 + e2 + e3 + e4
s2 = e1 - e2
s3 = e4 - e3
s4 = (e1 + e2) - (e3 + e4)

3) if the primary mode signal is 1 and the permutation signal is 0:

s1 = e1 + A.e3 - B.e4
s2 = e2 + B.e3 + A.e4
s3 = e1 - A.e3 + B.e4
s4 = -e2 + B.e3 + A.e4

4) if the primary mode signal is 1 and the permutation signal is 1:

s1 = e1 - A.e3 + B.e4
s2 = -e2 + B.e3 + A.e4
s3 = e1 - A.e3 - B.e4
s4 = e2 + B.e3 + A.e4

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14. The calculation method according to claim 10, characterized in that, in transformation step E_p, each calculation block comprises:

- 2^p-1 internal butterflies and a peripheral butterfly for the even values of index p as well as for the last transformation step if μ is even, and
- 2^p-1 internal butterflies, otherwise.

15. The calculation method according to claim 13, characterized in that the secondary mode signal is 1 if the peripheral butterfly is used for the last transformation step with μ being odd, and otherwise 0.

16. The calculation method according to claim 15, characterized in that, for four input samples e1, e2, e3, and e4, and for a complex coefficient $W^S=A+j.B$, the butterfly delivers the following output samples s1, s2, s3, and s4

1) if primary mode, secondary mode and permutation signals are 0:

s1 = e1 + e2 + e3 + e4

s2 = e1 - e2

s3 = e4 - e3

s4 = (e1 + e2) - (e3 + e4)

2) if the primary mode signal is 0 and the secondary mode signal is 1:

s1 = e1 + e4

s2 = e2

s3 = e3

s4 = e1 - e4

3) if the primary mode signal is 0 and the permutation signal is 1:

s1 = (e3 + e4) - (e1 + e2)

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$s2 = e1 - e2$

$s3 = e4 - e3$

$s4 = e1 + e2 + e3 + e4$

4) if the primary mode signal is 1 and the permutation signal is 0:

$s1 = e1 + A.e3 - B.e4$

$s2 = e2 + B.e3 + A.e4$

$s3 = e1 - A.e3 + B.e4$

$s4 = -e2 + B.e3 + A.e4$

5) if the primary mode signal is 1 and the permutation signal is 1:

$s1 = e1 - A.e3 + B.e4$

$s2 = -e2 + B.e3 + A.e4$

$s3 = e1 + A.e3 - B.e4$

$s4 = e2 + B.e3 + A.e4$

17. The calculation method according to claim 10, characterized in that, in transformation step E_p, each calculation block comprises:

- 2^p-1 internal butterflies and a peripheral butterfly for the even values of index p, and
- 2^p-1 internal butterflies, otherwise.

18. The calculation method according to claim 17, characterized in that the secondary mode signal is 1 if the peripheral butterfly is used for the first transformation step with μ being even, and otherwise 0.

19. The calculation method according to claim 18, characterized in that, for four input samples e1, e2, e3, and e4, and for a complex coefficient $W^s=A+j.B$, the butterfly delivers the following output samples s1, s2, s3, and s4

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1) if the primary mode signal is 0 and the secondary mode signal is 1:

$$s1 = e1 + e2$$
$$s2 = e1 - e2$$
$$s3 = e4 - e3$$
$$s4 = e3 + e4$$

2) if primary mode, secondary mode and permutation signals are 0:

$$s1 = e1 + e2 + e3 + e4$$
$$s2 = e1 - e2$$
$$s3 = e4 - e3$$
$$s4 = (e1 + e2) - (e3 + e4)$$

3) if the primary mode and secondary mode signals are 0 and the permutation signal is 1:

$$s1 = (e3 + e4) - (e1 + e2)$$
$$s2 = e1 - e2$$
$$s3 = e4 - e3$$
$$s4 = e1 + e2 + e3 + e4$$

4) if the primary mode signal is 1 and the permutation signal is 0:

$$s1 = e1 + A.e3 - B.e4$$
$$s2 = e2 + B.e3 + A.e4$$
$$s3 = e1 - A.e3 + B.e4$$
$$s4 = -e2 + B.e3 + A.e4$$

5) if the primary mode signal is 1 and the permutation signal is 1:

$$s1 = e1 - A.e3 + B.e4$$
$$s2 = -e2 + B.e3 + A.e4$$
$$s3 = e1 + A.e3 - B.e4$$
$$s4 = e2 + B.e3 + A.e4$$

20. The calculation method according to claim 8, characterized in that the butterflies for implementing the transformation steps are all of the same type and have

- four inputs for receiving input samples and four outputs for providing output samples,
- four additional inputs, respectively primary mode, secondary mode, permutation, and coefficient inputs,

in order to selectively apply different transformation operations to the input samples, each operation being determined by the values assigned to the primary mode, secondary mode, permutation signals, and a coefficient admitted on said corresponding additional inputs,

and in that the final step furthermore performs an addition and subtraction between the first and the last output sample provided in the last transformation step.

21. The calculation method according to claim 20, characterized in that, in transformation step E_p, each calculation block comprises one peripheral butterfly and 2^P-1 internal butterflies.

22. The calculation method according to claim 21, characterized in that, for four input samples e1, e2, e3, and e4, and for a complex coefficient $W^S=A+j.B$, the butterfly delivers the following output samples s1, s2, s3, and s4

1) if the primary mode signal is 0:

$s1 = e1 + e2$

$s2 = e1 - e2$

$s3 = e4 - e3$

s4 = e3 + e4

2) if the primary mode signal is 1 and the permutation signal is 0:

s1 = e1 + A.e3 - B.e4

s2 = e2 + B.e3 + A.e4

s3 = e1 - A.e3 + B.e4

s4 = -e2 + B.e3 + A.e4

3) if the primary signal is 1 and the permutation signal is 1:

s1 = e1 - A.e3 + B.e4

s2 = -e2 + B.e3 + A.e4

s3 = e1 + A.e3 -B.e4

s4 = e2 + B.e3 + A.e4

23. The calculation method according to claim 9 or 20, characterized in that the first and second binary addresses of μ bits are generated for each butterfly, each binary address corresponding to the rank of an input sample of said butterfly and the second binary address being greater than the first binary address.

24. The calculation method according to claim 23, characterized in that said first and second binary addresses are consecutive and an internal butterfly is involved.

25. The calculation method according to claim 23 or 24, characterized in that, if a peripheral butterfly is involved, the p+2 low-order bits of the first address are equal to 0, and the p+2 low-order bits of the second address form a number equal to $2^{p+1}-1$.

26. The calculation method according to claim 24 or 25, characterized in that the address of the two other samples to be applied to the inputs of the

butterfly, be they peripheral or internal, are obtained by inverting the (p+2) low-order bits of said first and second produced addresses.

27. The calculation method according to claim 26, characterized in that even-numbered address samples and odd-numbered address samples are stored in two separate memories.

28. The calculation method according to claim 25, characterized in that the value of the parameter s of the coefficient W^s assigned to an internal butterfly in transformation step E_p is coded by $\mu-2$ bits, and is:

- if $p+1=\mu-2$, the number formed by the p+1 low-order bits of the second binary address produced for said internal butterfly,
- if $p+1<\mu-2$, the number formed by the p+1 low-order bits of the second binary address produced for said internal butterfly, followed by $\mu-p-3$ zero bits at the end of the number,
- if $p+1>\mu-2$, the number formed by the p+1 low-order bits of the second binary address produced for said internal butterfly, minus its $\mu-p-1$ low-order bits.

29. The calculation method according to claim 4, in turn dependent on claim 3, in turn dependent on claim 2, characterized in that in each transformation step E_p, the butterflies are distributed among 2^P calculation blocks,

in that each calculation block comprises one peripheral butterfly and $N/2^{p+2}-1$ internal butterflies,

in that the peripheral butterfly of the rank α calculation block in transformation step E_β transforms

the input samples of rank $2^{\mu-\beta}\alpha$, $2^{\mu-\beta}\alpha+2^{\mu-\beta-1}-1$, $2^{\mu-\beta}\alpha+2^{\mu-\beta-1}$, $2^{\mu-\beta}\alpha+2^{\mu-\beta}-1$ into output samples of the same rank,

and in that the internal rank τ butterfly of the rank α calculation block in transformation step E_β transforms the input samples of rank $2^{\mu-\beta}\alpha+2\tau+1$, $2^{\mu-\beta}\alpha+2\tau+2$, $2^{\mu-\beta}\alpha+2^{\mu-\beta-1}-2\tau-3$, $2^{\mu-\beta}\alpha+2^{\mu-\beta}-2\tau-2$ into output samples of the same rank.

30. The calculation method according to claim 29, characterized in further comprising a final step of modifying the sequence of the output samples provided in the last transformation step and classifying them in ascending order of index n.

31. The calculation method according to claim 29 or 30, characterized in that each butterfly is assigned a coefficient W^s, whereon the calculation inside the butterfly is based, said coefficient being equal to $e^{-j(2\pi s/N)}$ with $s \in [0..N/4-1]$ for a fast Fourier transform and is equal to $e^{j(2\pi s/N)}$ with $s \in [0..N/4-1]$ for an inverse fast Fourier transform.

32. Calculation method according to claim 31, characterized in that the internal rank τ butterfly of the rank α calculation block in transformation step E_β is assigned coefficient W^δ with $\delta = (\tau+1).2^\beta$.

33. The calculation method according to claim 32, characterized in that the butterflies for implementing the transformation steps are all of the same type and have

- four inputs for receiving input samples and four outputs for providing output samples,

- four additional inputs, respectively primary mode, secondary mode, permutation, and coefficient inputs,

in order to selectively apply different transformation operations to the input samples, each operation being determined by the values assigned to the primary mode, secondary mode, permutation signals, and a coefficient admitted on said corresponding additional inputs.

34. The calculation method according to claim 33, characterized in that, for each butterfly, the primary mode signal is 0 for a peripheral butterfly and 1 for an internal butterfly,

in that the permutation signal is 0 for the even rank calculation blocks, including rank 0, and 1 for the odd values.

35. The calculation method according to claim 31 or 34, characterized in that the secondary mode signal is 1 if the butterfly, be it peripheral or internal, is used for the first transformation step, and otherwise 0.

36. The calculation method according to claim 35, characterized in that, for four input samples e1, e2, e3, and e4, and for a complex coefficient $W^S=A+j.B$, the butterfly delivers the following output samples s1, s2, s3, and s4

1) if the primary mode and secondary mode signals are 0:

$$s1 = (e1 + e2)/2$$

$$s2 = (e1 - e2)/2$$

$$s3 = (e4 - e3)/2$$

$s4 = (e3 + e4)/2$

2) if the primary mode signal is 0 and the secondary mode signal is 1:

$s1 = [(e1+e4)/2-e2]/2$

$s2 = [(e1+e4)/2-e2]/2$

$s3 = [e3-(e1-e4)/2]/2$

$s4 = [e3+(e1+e4)/2]/2$

3) if the primary mode signal is 1 and the permutation signal is 0:

$s1 = (e1+e3)/2$

$s2 = (e2+e4)/2$

$s3 = [(e1-e3).A - (e2+e4).B]/2$

$s4 = [-(e1-e3).B + (e2+e4).A]/2$

4) if the primary mode signal is 1 and the permutation signal is 1:

$s1 = [(e1-e3).A - (e2+e4).B]/2$

$s2 = [-(e1-e3).B + (e2+e4).A]/2$

$s3 = (e1+e3)/2$

$s4 = (e2-e4)/2$

37. The calculation method according to claim 33, characterized in that the first and second binary addresses of μ bits are generated for each butterfly, each binary address corresponding to the rank of an input sample of said butterfly and the second binary address being greater than the first binary address.

38. The calculation method according to claim 37, characterized in that said first and second binary addresses are consecutive and an internal butterfly is involved.

39. The calculation method according to claim 37 or 38, characterized in that, if a peripheral butterfly

is involved, the μ-p low-order bits of the first address are equal to 0, and the μ-p low-order bits of the second address form a number equal to $N/2^{p+1}-1$.

40. The calculation method according to claim 38 or 39, characterized in that the address of the two other samples to be applied to the inputs of the butterfly are obtained by inverting the μ-p low-order bits of both produced addresses.

41. The calculation method according to claim 40, characterized in that even-numbered address samples and odd-numbered address samples are stored in two separate memories.

42. The calculation method according to claim 41, characterized in that the value of the parameter s of the coefficient W^s assigned to an internal butterfly in transformation step E_p is coded by μ-2 bits, and is:

- if μ-p-1=μ-2, the number formed by the μ-p-1 low-order bits of the second address produced for said internal butterfly,

- if μ-p-1<μ-2, the number formed by the μ-p-1 low-order bits of the second address produced for said internal butterfly, followed by p-1 zero bits at the end of the number,

- if μ-p-1>μ-2, the number formed by the μ-p-1 low-order bits of the second address produced for said internal butterfly, minus its p+1 low-order bits.